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Interaction between a screw dislocation and a viscoelastic piezoelectric bimaterial interface

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Abstract

The complex variable method is employed to derive analytical solutions for the interaction between a piezoelectric screw dislocation and a Kelvin-type viscoelastic piezoelectric bimaterial interface. Through analytical continuation, the original boundary value problem can be reduced to an inhomogeneous first-order partial differential equation for a single function of location $z = x + iy$ and time t defined in the lower half-plane, which is free of the screw dislocation. Once the initial, steady-state and far-field conditions are known, the solution to the first order differential equation can be obtained. From the solved function, explicit expressions are then derived for the stresses, strains, electric fields and electric displacements induced by the piezoelectric screw dislocation. Also presented is the image force acting on the screw dislocation due to its interaction with the Kelvin-type viscoelastic interface. The derived solutions are verified by comparing with existing solutions for the simplified cases, and various interesting features are observed, particularly for those associated with the image force.

Keywords: Viscoelastic interface; Piezoelectric bimaterial; Dislocation; Complex variable method

1. Introduction

Interaction between dislocations and interface is an intriguing topic in micromechanics. In the majority of the studies the interface is simplified to be perfect (Head, 1953; Dundurs and Mura, 1964; Kelly et al., 1993, 1994; Wang and Sudak, 2006) or is modeled by a linear spring layer of vanishing thickness (see, for example, Wang and Shen, 2002; Fan and Wang, 2003a; Sudak, 2003; Sudak and Wang, 2006; Wang and Sudak, 2007 among others). Recently Fan and Wang (2003b) considered the interaction of a straight screw dislocation with a Kelvin- or Maxwell-type viscoelastic interface by means of Laplace and Fourier transformations. The interface considered by Fan and Wang (2003b) is modeled by linear spring and dashpot. They presented explicit expressions of the image force on the dislocation due to its interaction with the viscoelastic interface. In Fan and Wang (2003b), the two half-planes of the bimaterial are purely elastic, and the expressions of the stresses

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induced by the dislocation are implicit in the sense that inverse Fourier transformation needs to be carried out [see Eqs. (2.22)–(2.25) in Fan and Wang (2003b) for the Kelvin-type interface]. In fact thin viscoelastic damping layers can be artificially introduced into smart/intelligent structures to produce higher and tailorable damping in vibration control (Baz, 1993; Shen, 1994; Haung et al., 1996; Liao and Wang, 1997; Lee and Kim, 2001; Sun and Tong, 2003). To the best of the authors' knowledge, the viscoelastic behavior of the interface in piezoelectric composite has not yet been touched despite the fact that existence of a viscoelastic interface will definitely influence the response of the piezoelectric composite.

In this investigation we address in detail a screw dislocation in a piezoelectric (or more specifically ferroelectric) bimaterial with a Kelvin-type viscoelastic imperfect interface by employing the complex variable method. Both the electroded and unelectroded cases for the interface are discussed. It is found that the complex variable method is very suitable to study the interaction of a screw dislocation with a Kelvin-type viscoelastic interface between two bonded piezoelectric half-planes. The explicit expressions of the stresses, strains, electric displacements and electric fields induced by the screw dislocation are obtained. Concise expressions of the image force on the screw dislocation are also presented, including further discussion on some special cases involved.

2. Basic formulations

In a fixed rectangular coordinate system (x, y, z), we consider a screw dislocation located at a point $x=0$, $y=\delta$, ($\delta>0$) in the upper piezoelectric (or more specifically ferroelectric) half-plane of a piezoelectric bimaterial, as shown in Fig. 1. Both the upper piezoelectric half-plane $y\geqslant 0$, denoted by #1, and the lower piezoelectric half-plane $y\leqslant 0$, denoted by #2, are transversely isotropic with the poling direction parallel to the z-axis. The screw dislocation is assumed to be straight and infinitely long in the z-direction, experiencing a displacement jump b and an electric potential jump $\Delta\phi$ across the slip plane. The bimaterial interface $y=0$ considered in this investigation possesses Kelvin-type viscoelasticity.

For the problem described above, the governing equations and constitutive equations can be simplified considerably as follows

– Governing field equations:

$$\sigma_{zx,x}+\sigma_{zy,y}=0,\quad D_{x,x}+D_{y,y}=0, \tag{1}$$

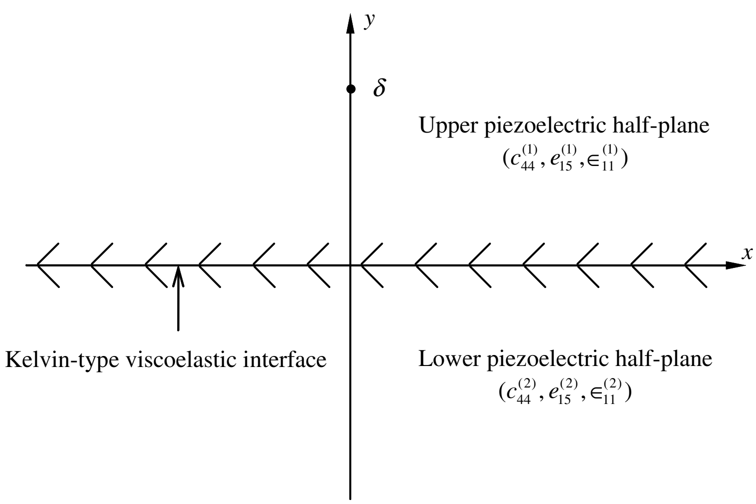

Fig. 1. A piezoelectric screw dislocation located at $x=0$ and $y=\delta(\delta>0)$ in the upper piezoelectric half-plane which is bonded to a lower piezoelectric half-plane via a Kelvin-type viscoelastic interface $y=0$.

– Electric field-electric potential relations:

$$E_x = -\phi_{,x}, \quad E_y = -\phi_{,y}, \tag{2}$$

– Linear, piezoelectric constitutive equations:

$$\begin{bmatrix} \sigma_{zy} \\ D_y \end{bmatrix} = \begin{bmatrix} c_{44} & -e_{15} \\ e_{15} & \in_{11} \end{bmatrix} \begin{bmatrix} w_{,y} \\ E_y \end{bmatrix}, \tag{3a}$$

$$\begin{bmatrix} \sigma_{zx} \\ D_x \end{bmatrix} = \begin{bmatrix} c_{44} & -e_{15} \\ e_{15} & \in_{11} \end{bmatrix} \begin{bmatrix} w_{,x} \\ E_x \end{bmatrix}, \tag{3b}$$

where a comma followed by x (or y) denotes partial derivatives with respect to x (or y); σ_{zx}, σ_{zy} are the shear stress components; D_x, D_y are the electric displacement components; E_x, E_y are the electric fields; w is the out-of-plane displacement; ϕ is the electric potential; c_{44}, e_{15}, and $\in_{11}$ are, respectively, the elastic modulus, piezoelectric constant, and dielectric permittivity. In this paper the piezoelectrically stiffened elastic constant $\tilde{c}_{44} = c_{44} + e_{15}^2/\in_{11}$ will be also used. In Eq. (1) we have neglected the inertial effect of the piezoelectric material due to the fact that the viscoelastic response comes from the interface only and that the deformation of the piezoelectric bimaterial is assumed to be in a quasi-static state (Fan and Wang, 2003b; Ang and Fan, 2004; Yan et al., 2006).

The displacement and electric potential can be expressed in terms of two analytic functions $f_1(z,t)$ and $f_2(z,t)$, $(z = x + \mathrm{i}y)$ as

$$w = \mathrm{Im}\{f_1(z,t)\}, \quad \phi = \mathrm{Im}\{f_2(z,t)\}. \tag{4}$$

Since the viscoelastic interface exhibits the time effect, the two analytic functions $f_1(z,t)$ and $f_2(z,t)$ depend not only on the complex variable z but also on the time t. In terms of the two analytic functions, the strains, electric fields, stresses and electric displacements can be expressed as

$$\begin{aligned} &\gamma_{zy} + \mathrm{i}\gamma_{zx} = \frac{\partial f_1(z,t)}{\partial z}, \quad -E_y - \mathrm{i}E_x = \frac{\partial f_2(z,t)}{\partial z}, \\ &\sigma_{zy} + \mathrm{i}\sigma_{zx} = c_{44}\frac{\partial f_1(z,t)}{\partial z} + e_{15}\frac{\partial f_2(z,t)}{\partial z}, \quad D_y + \mathrm{i}D_x = e_{15}\frac{\partial f_1(z,t)}{\partial z} - \in_{11}\frac{\partial f_2(z,t)}{\partial z}, \end{aligned} \tag{5}$$

where the strains γ_{zx} and γ_{zy} are related to the out-of-plane displacement w through

$$\gamma_{zx} = w_{,x}, \quad \gamma_{zy} = w_{,y}. \tag{6}$$

In this paper, the superscripts (1) and (2) will be used to denote, respectively, the physical quantities in the upper and lower half-planes. The two analytic functions, as defined in Eq. (4), are denoted by $g_1(z,t)$ and $g_2(z,t)$ in the upper half-plane and by $h_1(z,t)$ and $h_2(z,t)$ in the lower half-plane. Both the electroded and unelectroded interfaces will be considered.

The continuity conditions on an unelectroded Kelvin-type viscoelastic interface are given by (Fan and Wang, 2003b; Fan et al., 2006)

$$\left.\begin{aligned} &\sigma_{zy}^{(1)} = \sigma_{zy}^{(2)}, \quad D_y^{(1)} = D_y^{(2)}, \quad \phi^{(1)} = \phi^{(2)}, \\ &\sigma_{zy}^{(2)} = k[w^{(1)} - w^{(2)}] + \eta\frac{\partial}{\partial t}[w^{(1)} - w^{(2)}], \end{aligned}\right. \text{on } y = 0, \tag{7}$$

where k is the spring constant of the interface and η the viscosity coefficient. Eq. (7) is similar to that adopted by Fan et al. (2006) except that a linear dashpot is added to our model. In this model a linear spring and a linear dashpot are parallel-connected (Fan and Wang, 2003b). The above expression implies that the interface is dielectrically perfect, i.e., both the normal electric displacement and electric potential are continuous across the interface.

The continuity conditions on an electroded Kelvin-type viscoelastic interface are given by (Fan and Wang, 2003b; Fan et al., 2006)

$$\begin{aligned}&\sigma_{zy}^{(1)}=\sigma_{zy}^{(2)},\quad \phi^{(1)}=\phi^{(2)}=0,\\&\sigma_{zy}^{(2)}=k[w^{(1)}-w^{(2)}]+\eta\frac{\partial}{\partial t}[w^{(1)}-w^{(2)}],\end{aligned}\quad \text{on } y=0. \tag{8}$$

In this model a grounded electrode is placed on the interface.

3. Full-field solutions

3.1. Unelectroded Kelvin-type viscoelastic interface

The continuity conditions Eq. (7) on an unelectroded Kelvin-type viscoelastic interface can be equivalently expressed in terms of $g_1(z,t)$, $g_2(z,t)$ defined in the upper half-plane and $h_1(z,t)$, $h_2(z,t)$ defined in the lower half-plane as follows

$$\begin{aligned}&c_{44}^{(1)}[g_1^+(x,t)+\bar{g}_1^-(x,t)]+e_{15}^{(1)}[g_2^+(x,t)+\bar{g}_2^-(x,t)]=c_{44}^{(2)}[h_1^-(x,t)+\bar{h}_1^+(x,t)]+e_{15}^{(2)}[h_2^-(x,t)+\bar{h}_2^+(x,t)],\\&e_{15}^{(1)}[g_1^+(x,t)+\bar{g}_1^-(x,t)]-\in_{11}^{(1)}[g_2^+(x,t)+\bar{g}_2^-(x,t)]=e_{15}^{(2)}[h_1^-(x,t)+\bar{h}_1^+(x,t)]-\in_{11}^{(2)}[h_2^-(x,t)+\bar{h}_2^+(x,t)],\\&g_2^+(x,t)-\bar{g}_2^-(x,t)=h_2^-(x,t)-\bar{h}_2^+(x,t),\\&k[g_1^+(x,t)-\bar{g}_1^-(x,t)-h_1^-(x,t)+\bar{h}_1^+(x,t)]+\eta\frac{\partial}{\partial t}[g_1^+(x,t)-\bar{g}_1^-(x,t)-h_1^-(x,t)+\bar{h}_1^+(x,t)]\\&\quad=\mathrm{i}c_{44}^{(2)}\frac{\partial}{\partial x}[h_1^-(x,t)+\bar{h}_1^+(x,t)]+\mathrm{i}e_{15}^{(2)}\frac{\partial}{\partial x}[h_2^-(x,t)+\bar{h}_2^+(x,t)].\quad \text{on } y=0.\end{aligned}\tag{9}$$

It follows from (9)$_{1-3}$ that the three functions $g_1(z,t)$, $g_2(z,t)$ and $\bar{h}_2(z,t)$ defined in the upper half-plane can be expressed in terms of one single function $\bar{h}_1(z,t)$ also defined in the upper half-plane, as

$$\begin{aligned}g_1(z,t)&=\frac{\tilde{c}_{44}^{(2)}\in_{11}^{(2)}+c_{44}^{(2)}\in_{11}^{(1)}+e_{15}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}\bar{h}_1(z,t)+g_{10}(z)-\bar{g}_{10}(z)+\frac{2\left(\in_{11}^{(1)}e_{15}^{(2)}-\in_{11}^{(2)}e_{15}^{(1)}\right)}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}\bar{g}_{20}(z),\\g_2(z,t)&=\frac{c_{44}^{(2)}e_{15}^{(1)}-c_{44}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}\bar{h}_1(z,t)+g_{20}(z)+\frac{c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}-\tilde{c}_{44}^{(1)}\in_{11}^{(1)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}\bar{g}_{20}(z),\\\bar{h}_2(z,t)&=\frac{c_{44}^{(1)}e_{15}^{(2)}-c_{44}^{(2)}e_{15}^{(1)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}\bar{h}_1(z,t)+\frac{2\tilde{c}_{44}^{(1)}\in_{11}^{(1)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}\bar{g}_{20}(z),\end{aligned}\tag{10}$$

where $g_{10}(z)=\frac{b}{2\pi}\ln(z-\mathrm{i}\delta)$ and $g_{20}(z)=\frac{\Delta\phi}{2\pi}\ln(z-\mathrm{i}\delta)$ are the complex potentials for a screw dislocation located at $z=\mathrm{i}\delta$ in a homogeneous material.

Similarly the three functions $\bar{g}_1(z,t)$, $\bar{g}_2(z,t)$ and $h_2(z,t)$ defined in the lower half-plane can be expressed in terms of one single function $h_1(z,t)$ also defined in the lower half-plane, as

$$\begin{aligned}\bar{g}_1(z,t)&=\frac{\tilde{c}_{44}^{(2)}\in_{11}^{(2)}+c_{44}^{(2)}\in_{11}^{(1)}+e_{15}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}h_1(z,t)+\bar{g}_{10}(z)-g_{10}(z)+\frac{2\left(\in_{11}^{(1)}e_{15}^{(2)}-\in_{11}^{(2)}e_{15}^{(1)}\right)}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}g_{20}(z),\\\bar{g}_2(z,t)&=\frac{c_{44}^{(2)}e_{15}^{(1)}-c_{44}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}h_1(z,t)+\bar{g}_{20}(z)+\frac{c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}-\tilde{c}_{44}^{(1)}\in_{11}^{(1)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}g_{20}(z),\\h_2(z,t)&=\frac{c_{44}^{(1)}e_{15}^{(2)}-c_{44}^{(2)}e_{15}^{(1)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}h_1(z,t)+\frac{2\tilde{c}_{44}^{(1)}\in_{11}^{(1)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}g_{20}(z).\end{aligned}\tag{11}$$

Substituting Eqs. (10) and (11) into Eq. (9)$_4$, we finally arrive at

$$
\begin{aligned}
&k\frac{\left(\tilde{c}_{44}^{(1)}+c_{44}^{(2)}\right)\in_{11}^{(1)}+(c_{44}^{(1)}+\tilde{c}_{44}^{(2)})\in_{11}^{(2)}+2e_{15}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}\bar{h}_1^{+}(x,t)-\mathrm{i}\frac{\tilde{c}_{44}^{(1)}c_{44}^{(2)}\in_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}\frac{\partial\bar{h}_1^{+}(x,t)}{\partial x}\\
&\quad+\eta\frac{\left(\tilde{c}_{44}^{(1)}+c_{44}^{(2)}\right)\in_{11}^{(1)}+\left(c_{44}^{(1)}+\tilde{c}_{44}^{(2)}\right)\in_{11}^{(2)}+2e_{15}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}\frac{\partial\bar{h}_1^{+}(x,t)}{\partial t}-2k\bar{g}_{10}(x)-\frac{2k\left(\in_{11}^{(2)}e_{15}^{(1)}-\in_{11}^{(1)}e_{15}^{(2)}\right)}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}\bar{g}_{20}(x)\\
&\quad-\frac{2\mathrm{i}\tilde{c}_{44}^{(1)}\in_{11}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}\bar{g}'_{20}(x)=k\frac{\left(\tilde{c}_{44}^{(1)}+c_{44}^{(2)}\right)\in_{11}^{(1)}+\left(c_{44}^{(1)}+\tilde{c}_{44}^{(2)}\right)\in_{11}^{(2)}+2e_{15}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}h_1^{-}(x,t)\\
&\quad+\mathrm{i}\frac{\tilde{c}_{44}^{(1)}c_{44}^{(2)}\in_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}\frac{\partial h_1^{-}(x,t)}{\partial x}+\eta\frac{\left(\tilde{c}_{44}^{(1)}+c_{44}^{(2)}\right)\in_{11}^{(1)}+\left(c_{44}^{(1)}+\tilde{c}_{44}^{(2)}\right)\in_{11}^{(2)}+2e_{15}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}\frac{\partial h_1^{-}(x,t)}{\partial t}\\
&\quad-2kg_{10}(x)-\frac{2k\left(\in_{11}^{(2)}e_{15}^{(1)}-\in_{11}^{(1)}e_{15}^{(2)}\right)}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}g_{20}(x)+\frac{2\mathrm{i}\tilde{c}_{44}^{(1)}\in_{11}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}g'_{20}(x).\quad \text{on } y=0.
\end{aligned}
\tag{12}
$$

It is apparent that the left-hand side of Eq. (12) is analytic in the upper half-plane, whilst the right-hand side of Eq. (12) is analytic in the lower half-plane. Consequently the continuity condition in Eq. (12) implies that the left- and right-hand sides of Eq. (12) are identically zero in the upper and lower half-planes, respectively. It follows that

$$
-\mathrm{i}\chi h_1(z,t)+\frac{\partial h_1(z,t)}{\partial z}-\mathrm{i}\gamma\frac{\partial h_1(z,t)}{\partial t}=-\mathrm{i}\chi\alpha\ln(z-\mathrm{i}\delta)-\frac{\beta}{z-\mathrm{i}\delta},\quad y\leqslant 0 \tag{13}
$$

or equivalently

$$
-\mathrm{i}\chi\frac{\partial h_1(z,t)}{\partial z}+\frac{\partial^2 h_1(z,t)}{\partial z^2}-\mathrm{i}\gamma\frac{\partial^2 h_1(z,t)}{\partial z\partial t}=-\frac{\mathrm{i}\chi\alpha}{z-\mathrm{i}\delta}+\frac{\beta}{(z-\mathrm{i}\delta)^2},\quad y\leqslant 0, \tag{14}
$$

where

$$
\begin{aligned}
\chi&=k\frac{(\tilde{c}_{44}^{(1)}+c_{44}^{(2)})\in_{11}^{(1)}+(c_{44}^{(1)}+\tilde{c}_{44}^{(2)})\in_{11}^{(2)}+2e_{15}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}c_{44}^{(2)}\in_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)}},\\
\gamma&=\eta\frac{(\tilde{c}_{44}^{(1)}+c_{44}^{(2)})\in_{11}^{(1)}+(c_{44}^{(1)}+\tilde{c}_{44}^{(2)})\in_{11}^{(2)}+2e_{15}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}c_{44}^{(2)}\in_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)}},
\end{aligned}
\tag{15}
$$

and

$$
\begin{aligned}
\alpha&=\frac{(\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)})b+(\in_{11}^{(2)}e_{15}^{(1)}-\in_{11}^{(1)}e_{15}^{(2)})\Delta\phi}{\pi\left[(\tilde{c}_{44}^{(1)}+c_{44}^{(2)})\in_{11}^{(1)}+(c_{44}^{(1)}+\tilde{c}_{44}^{(2)})\in_{11}^{(2)}+2e_{15}^{(1)}e_{15}^{(2)}\right]},\\
\beta&=\frac{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}e_{15}^{(2)}\Delta\phi}{\pi(\tilde{c}_{44}^{(1)}c_{44}^{(2)}\in_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)})}.
\end{aligned}
\tag{16}
$$

Eq. (14) is an inhomogeneous first-order partial differential equation for function $\frac{\partial h_1(z,t)}{\partial z}$. It is noted that, at $t=0$ when the piezoelectric screw dislocation is just introduced into the upper piezoelectric half-plane, the displacement across the interface has no time to experience any jump due to the dashpot. Therefore the displacement is continuous across the interface at $t=0$ (i.e., the interface is perfect when $t=0$). In order words, the following initial condition for $\frac{\partial h_1(z,t)}{\partial z}$ holds

$$
\frac{\partial h_1(z,0)}{\partial z}=\frac{\alpha}{z-\mathrm{i}\delta}. \tag{17}
$$

When $t\to\infty$, on the other hand, the interface should be at a steady state and there is no time effect. In this case it follows from Eq. (13) that

$$-\mathrm{i}\chi h_1(z,\infty)+\frac{\partial h_1(z,\infty)}{\partial z}=-\mathrm{i}\chi\alpha\ln(z-\mathrm{i}\delta)-\frac{\beta}{z-\mathrm{i}\delta}. \tag{18}$$

The solution to the above differential equation can be readily derived as (for more details see Sudak and Wang, 2006)

$$\frac{\partial h_1(z,\infty)}{\partial z}=\mathrm{i}\chi(\alpha+\beta)\exp[\mathrm{i}\chi(z-\mathrm{i}\delta)]E_1[\mathrm{i}\chi(z-\mathrm{i}\delta)]-\frac{\beta}{z-\mathrm{i}\delta}, \tag{19}$$

where the exponential integral is defined by

$$E_1(z)=-\int_{\infty}^{z}\frac{e^{-q}}{q}\,\mathrm{d}q. \tag{20}$$

In addition,

$$\frac{\partial h_1(z,t)}{\partial z}\to 0 \quad \text{as } z\to\infty \tag{21}$$

due to the fact that at the far field the stresses and electric displacements should approach zero. In view of the initial state Eq. (17), the steady state Eq. (19) and the far-field condition Eq. (21), the solution to Eq. (13) or (14) can be derived to be

$$\begin{aligned}\frac{\partial h_1(z,t)}{\partial z}&=\mathrm{i}\chi(\alpha+\beta)\exp[\mathrm{i}\chi(z-\mathrm{i}\delta)]\{E_1[\mathrm{i}\chi(z-\mathrm{i}\delta)]-E_1[\mathrm{i}\chi(z-\mathrm{i}\delta-\mathrm{i}t/\gamma)]\}\\&\quad+\frac{(\alpha+\beta)\exp(-\chi t/\gamma)}{z-\mathrm{i}\delta-\mathrm{i}t/\gamma}-\frac{\beta}{z-\mathrm{i}\delta}.\end{aligned} \tag{22}$$

It can be shown solution (22) satisfies the conditions Eqs. (17), (19) and (21). Here we shall also mention that the term $\mathrm{i}\chi(\alpha+\beta)\exp[\mathrm{i}\chi(z-\mathrm{i}\delta)]E_1[\mathrm{i}\chi(z-\mathrm{i}\delta)]-\frac{\beta}{z-\mathrm{i}\delta}$ in Eq. (22) is a particular solution to Eq. (14) whilst $G_1(z,t)=\exp[\mathrm{i}\chi(z-\mathrm{i}\delta)]E_1[\mathrm{i}\chi(z-\mathrm{i}\delta-\mathrm{i}t/\gamma)]$ and $G_2(z,t)=\frac{\exp(-\chi t/\gamma)}{z-\mathrm{i}\delta-\mathrm{i}t/\gamma}$ in Eq. (22) are two homogeneous solutions to Eq. (14), namely,

$$-\mathrm{i}\chi G_j(z,t)+\frac{\partial G_j(z,t)}{\partial z}-\mathrm{i}\gamma\frac{\partial G_j(z,t)}{\partial t}=0,\quad j=1,2. \tag{23}$$

When $k=0$ (or $\chi\to 0$) for a viscous interface, it can be deduced from Eq. (22) that the expression of $\frac{\partial h_1(z,t)}{\partial z}$ for an unelectroded viscous interface is given by

$$\frac{\partial h_1(z,t)}{\partial z}=\frac{\alpha+\beta}{z-\mathrm{i}\delta-\mathrm{i}t/\gamma}-\frac{\beta}{z-\mathrm{i}\delta}, \tag{24}$$

which satisfies the partial differential equation

$$\frac{\partial^2 h_1(z,t)}{\partial z^2}-\mathrm{i}\gamma\frac{\partial^2 h_1(z,t)}{\partial z\partial t}=\frac{\beta}{(z-\mathrm{i}\delta)^2},\quad y\leqslant 0. \tag{25}$$

Once $\frac{\partial h_1(z,t)}{\partial z}$ is obtained, the expressions of $\frac{\partial h_2(z,t)}{\partial z}$, $\frac{\partial g_1(z,t)}{\partial z}$ and $\frac{\partial g_2(z,t)}{\partial z}$ can be found as follows

$$\frac{\partial h_2(z,t)}{\partial z}=\frac{c_{44}^{(1)}e_{15}^{(2)}-c_{44}^{(2)}e_{15}^{(1)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}\frac{\partial h_1(z,t)}{\partial z}+\frac{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}\Delta\phi}{\pi(\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)})(z-\mathrm{i}\delta)}, \tag{26}$$

$$\frac{\partial g_1(z,t)}{\partial z}=\frac{\tilde{c}_{44}^{(2)}\in_{11}^{(2)}+c_{44}^{(2)}\in_{11}^{(1)}+e_{15}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}\frac{\partial\bar{h}_1(z,t)}{\partial z}+\left[\frac{(\in_{11}^{(1)}e_{15}^{(2)}-\in_{11}^{(2)}e_{15}^{(1)})\Delta\phi}{\pi(\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)})}-\frac{b}{2\pi}\right]\frac{1}{z+\mathrm{i}\delta}+\frac{b}{2\pi(z-\mathrm{i}\delta)}, \tag{27}$$

$$\frac{\partial g_2(z,t)}{\partial z}=\frac{c_{44}^{(2)}e_{15}^{(1)}-c_{44}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}}\frac{\partial\bar{h}_1(z,t)}{\partial z}+\frac{(c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)}-\tilde{c}_{44}^{(1)}\in_{11}^{(1)})\Delta\phi}{2\pi(\tilde{c}_{44}^{(1)}\in_{11}^{(1)}+c_{44}^{(1)}\in_{11}^{(2)}+e_{15}^{(1)}e_{15}^{(2)})(z+\mathrm{i}\delta)}+\frac{\Delta\phi}{2\pi(z-\mathrm{i}\delta)}, \tag{28}$$

where the explicit expression of $\frac{\partial\bar{h}_1(z,t)}{\partial z}$ is given by

$$\frac{\partial \bar{h}_1(z,t)}{\partial z} = -\mathrm{i}\chi(\alpha+\beta)\exp[-\mathrm{i}\chi(z+\mathrm{i}\delta)]\{E_1[-\mathrm{i}\chi(z+\mathrm{i}\delta)] - E_1[-\mathrm{i}\chi(z+\mathrm{i}\delta+\mathrm{i}t/\gamma)]\} + \frac{(\alpha+\beta)\exp(-\chi t/\gamma)}{z+\mathrm{i}\delta+\mathrm{i}t/\gamma} - \frac{\beta}{z+\mathrm{i}\delta}. \tag{29}$$

With the expressions of $\frac{\partial g_1(z,t)}{\partial z}$ and $\frac{\partial g_2(z,t)}{\partial z}$ defined in the upper piezoelectric half-plane, $\frac{\partial h_1(z,t)}{\partial z}$ and $\frac{\partial h_2(z,t)}{\partial z}$ defined in the lower piezoelectric half-plane for an unelectroded Kelvin-type viscoelastic interface, the distributions of the strains, stresses, electric fields and electric displacements in the two half-planes can be obtained from Eq. (5). By using the Peach-Koehler formulation (Pak, 1990; Lee et al., 2000), the image force acting on the screw dislocation due to its interaction with the unelectroded Kelvin-type viscoelastic interface can be derived to be

$$\begin{aligned} F_y &= -\frac{p_{11}b^2 + 2p_{12}b\Delta\phi + p_{22}\Delta\phi^2}{4\pi\delta} \\ &\quad + \frac{q_{11}b^2 + 2q_{12}b\Delta\phi + q_{22}\Delta\phi^2}{4\pi\delta}\left\{g(\lambda) + \left(1+\frac{\tilde{t}}{2}\right)^{-1}\exp(-\lambda\tilde{t})\left[1 - g\left(\lambda + \frac{\lambda\tilde{t}}{2}\right)\right]\right\}, \\ F_x &= 0, \end{aligned} \tag{30}$$

where F_x and F_y are, respectively, the horizontal and vertical components of the image force; $\lambda = \delta\chi$ and $t_0 = \delta\gamma$ are, respectively, the interface "rigidity" and the relaxation time; $\tilde{t} = t/t_0$; $g(\eta) = 2\eta\,\exp(2\eta)E_1(2\eta)$, $(0 \leqslant g(\eta) \leqslant 1)$ is a monotonic function of η (see Fan and Wang, 2003a for more details) and the constants p_{ij}, q_{ij}, which are related to the material constants of the two piezoelectric half-planes, are defined as

$$\begin{aligned} p_{11} &= c_{44}^{(1)}, \quad p_{12} = e_{15}^{(1)}, \\ p_{22} &= \frac{e_{15}^{(1)2}}{c_{44}^{(1)}} + \frac{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}(c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)} - \tilde{c}_{44}^{(1)}c_{44}^{(2)}\in_{11}^{(1)})}{c_{44}^{(1)}(c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)} + \tilde{c}_{44}^{(1)}c_{44}^{(2)}\in_{11}^{(1)})}, \end{aligned} \tag{31}$$

$$\begin{aligned} q_{11} &= \frac{2(c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)} + c_{44}^{(2)}\tilde{c}_{44}^{(1)}\in_{11}^{(1)})}{(\tilde{c}_{44}^{(1)} + c_{44}^{(2)})\in_{11}^{(1)} + (c_{44}^{(1)} + \tilde{c}_{44}^{(2)})\in_{11}^{(2)} + 2e_{15}^{(1)}e_{15}^{(2)}}, \\ q_{12} &= \frac{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)} + c_{44}^{(1)}\in_{11}^{(2)} + e_{15}^{(1)}e_{15}^{(2)}} + \frac{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}e_{15}^{(2)} + \tilde{c}_{44}^{(2)}\in_{11}^{(2)}e_{15}^{(1)}}{(\tilde{c}_{44}^{(1)} + c_{44}^{(2)})\in_{11}^{(1)} + (c_{44}^{(1)} + \tilde{c}_{44}^{(2)})\in_{11}^{(2)} + 2e_{15}^{(1)}e_{15}^{(2)}} \\ &\quad + \frac{(c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)} + c_{44}^{(2)}\tilde{c}_{44}^{(1)}\in_{11}^{(1)})(\in_{11}^{(2)}e_{15}^{(1)} - \in_{11}^{(1)}e_{15}^{(2)})}{(\tilde{c}_{44}^{(1)}\in_{11}^{(1)} + c_{44}^{(1)}\in_{11}^{(2)} + e_{15}^{(1)}e_{15}^{(2)})\left[(\tilde{c}_{44}^{(1)} + c_{44}^{(2)})\in_{11}^{(1)} + (c_{44}^{(1)} + \tilde{c}_{44}^{(2)})\in_{11}^{(2)} + 2e_{15}^{(1)}e_{15}^{(2)}\right]}, \\ q_{22} &= \frac{2(\tilde{c}_{44}^{(1)}\in_{11}^{(1)}e_{15}^{(2)} + \tilde{c}_{44}^{(2)}\in_{11}^{(2)}e_{15}^{(1)})}{\tilde{c}_{44}^{(1)}\in_{11}^{(1)} + c_{44}^{(1)}\in_{11}^{(2)} + e_{15}^{(1)}e_{15}^{(2)}}\left[\frac{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}c_{44}^{(2)}\in_{11}^{(1)} + c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)}} + \frac{\in_{11}^{(2)}e_{15}^{(1)} - \in_{11}^{(1)}e_{15}^{(2)}}{(\tilde{c}_{44}^{(1)} + c_{44}^{(2)})\in_{11}^{(1)} + (c_{44}^{(1)} + \tilde{c}_{44}^{(2)})\in_{11}^{(2)} + 2e_{15}^{(1)}e_{15}^{(2)}}\right]. \end{aligned} \tag{32}$$

We remark that the present solution (the image force) includes a couple of special cases solved previously.

(i) If we ignore the piezoelectric effect for the two half-planes, i.e., $e_{15}^{(1)} = e_{15}^{(2)} = 0$, and let $\Delta\phi = 0$, then Eq. (30) is reduced to

$$F_y = -\frac{c_{44}^{(1)}b^2}{4\pi\delta}\left\{1 - \frac{2c_{44}^{(2)}}{c_{44}^{(1)} + c_{44}^{(2)}}\left[g(\lambda) + \left(1+\frac{\tilde{t}}{2}\right)^{-1}\exp(-\lambda\tilde{t})\left[1 - g\left(\lambda + \frac{\lambda\tilde{t}}{2}\right)\right]\right]\right\}, \tag{33}$$

where

$$\lambda = \delta k\frac{c_{44}^{(1)} + c_{44}^{(2)}}{c_{44}^{(1)}c_{44}^{(2)}}, \quad \text{and} \quad t_0 = \delta\eta\frac{c_{44}^{(1)} + c_{44}^{(2)}}{c_{44}^{(1)}c_{44}^{(2)}}. \tag{34}$$

Eq. (33) is just the result of Fan and Wang (2003b).

(ii) If we ignore the viscous effect of the interface, i.e., $\eta = 0$ or $\tilde{t} \to \infty$ and let $\Delta\phi = 0$, then Eq. (30) is reduced to

$$F_y = -\frac{c_{44}^{(1)} b^2}{4\pi\delta}\left[1 - \frac{2(c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)} + c_{44}^{(2)}\tilde{c}_{44}^{(1)}\in_{11}^{(1)})g(\lambda)}{c_{44}^{(1)}[(\tilde{c}_{44}^{(1)} + c_{44}^{(2)})\in_{11}^{(1)} + (c_{44}^{(1)} + \tilde{c}_{44}^{(2)})\in_{11}^{(2)} + 2e_{15}^{(1)}e_{15}^{(2)}]}\right], \tag{35}$$

which can be proved to be equivalent to Eq. (59) in Wang and Sudak (2007) for a mechanically compliant and dielectrically perfect interface.

(iii) If the two half-planes have the same material property and same poling direction, i.e., $c_{44}^{(1)} = c_{44}^{(2)} = c_{44}$, $e_{15}^{(1)} = e_{15}^{(2)} = e_{15}$, $\in_{11}^{(1)} = \in_{11}^{(2)} = \in_{11}$, then it follows from Eq. (30) that the image force on the dislocation is

$$F_y = -\frac{(c_{44}b + e_{15}\Delta\phi)^2}{4\pi c_{44}\delta}\left\{1 - g(\lambda) - \left(1 + \frac{\tilde{t}}{2}\right)^{-1}\exp(-\lambda\tilde{t})\left[1 - g\left(\lambda + \frac{\lambda\tilde{t}}{2}\right)\right]\right\} \leqslant 0, \tag{36}$$

which indicates that the dislocation is attracted to the interface. It shall be mentioned that in Eq. (36) $1 - g(\lambda) - (1 + \frac{\tilde{t}}{2})^{-1}\exp(-\lambda\tilde{t})[1 - g(\lambda + \frac{\lambda\tilde{t}}{2})] \geqslant 0$ (see Fig. 2 in Fan and Wang, 2003b). It can be found from expression (36) that if b and $\Delta\phi$ satisfy the relation $b = -\frac{e_{15}\Delta\phi}{c_{44}}$, the image force on the screw dislocation will be always zero.

(iv) If the two half-planes have the same material property but are poled in opposite directions, i.e., $c_{44}^{(1)} = c_{44}^{(2)} = c_{44}$, $e_{15}^{(1)} = -e_{15}^{(2)} = e_{15}$, $\in_{11}^{(1)} = \in_{11}^{(2)} = \in_{11}$, then it follows from Eq. (30) that the image force on the dislocation is

$$F_y = -\frac{(c_{44}b + e_{15}\Delta\phi)^2}{4\pi c_{44}\delta} + \frac{\tilde{c}_{44}b^2}{4\pi\delta}\left\{g(\lambda) + \left(1 + \frac{\tilde{t}}{2}\right)^{-1}\exp(-\lambda\tilde{t})\left[1 - g\left(\lambda + \frac{\lambda\tilde{t}}{2}\right)\right]\right\}. \tag{37}$$

Furthermore when b and $\Delta\phi$ satisfy the following inequality

$$\frac{b}{\Delta\phi} \leqslant -\frac{|e_{15}|}{c_{44}(\sqrt{1 + \tilde{c}_{44}/c_{44}} + e_{15}/|e_{15}|)} \quad \text{or} \quad \frac{b}{\Delta\phi} \geqslant \frac{|e_{15}|}{c_{44}(\sqrt{1 + \tilde{c}_{44}/c_{44}} - e_{15}/|e_{15}|)}, \tag{38}$$

or equivalently when $(c_{44}b + e_{15}\Delta\phi)^2 \geqslant \tilde{c}_{44}c_{44}b^2$, then it follows from Eq. (37) that

$$F_y \leqslant -\frac{\tilde{c}_{44}b^2}{4\pi\delta}\left\{1 - g(\lambda) - \left(1 + \frac{\tilde{t}}{2}\right)^{-1}\exp(-\lambda\tilde{t})\left[1 - g\left(\lambda + \frac{\lambda\tilde{t}}{2}\right)\right]\right\} \leqslant 0, \tag{39}$$

which implies that the dislocation is attracted to the interface.

(v) For a viscous interface we have $\lambda = 0$ and $g(0) = 0$, then it follows from Eq. (30) that

$$F_y = -\frac{p_{11}b^2 + 2p_{12}b\Delta\phi + p_{22}\Delta\phi^2}{4\pi\delta} + \frac{q_{11}b^2 + 2q_{12}b\Delta\phi + q_{22}\Delta\phi^2}{4\pi\delta}\left(1 + \frac{\tilde{t}}{2}\right)^{-1}. \tag{40}$$

When $\tilde{t} \to \infty$, Eq. (40) is reduced to

$$F_y = -\frac{p_{11}b^2 + 2p_{12}b\Delta\phi + p_{22}\Delta\phi^2}{4\pi\delta}, \tag{41}$$

which is the image force on a dislocation interacting with a traction-free and dielectrically perfect interface. If we further let $\in_{11}^{(2)} = e_{15}^{(2)} = 0$, then Eq. (41) becomes

$$F_y = -\frac{c_{44}^{(1)}b^2 + 2e_{15}^{(1)}b\Delta\phi - \in_{11}^{(1)}\Delta\phi^2}{4\pi\delta}, \tag{42}$$

which is just the result for a screw dislocation interacting with a traction-free and charge-free surface (Pak, 1990). On the other hand if we let $\in_{11}^{(2)} \to \infty$, then Eq. (41) becomes

$$F_y = -\frac{(c_{44}^{(1)}b + e_{15}^{(1)}\Delta\phi)^2 + \tilde{c}_{44}^{(1)}\in_{11}^{(1)}\Delta\phi^2}{4\pi c_{44}^{(1)}\delta} < 0, \tag{43}$$

which is the image force on a screw dislocation interacting with a traction-free and electroded surface. Different from the result for a dislocation interacting with a traction-free and charge-free surface (Pak, 1990 or see Eq. (42) in this paper), the piezoelectric screw dislocation is always attracted to a traction-free and electroded surface. In fact it can be found from Eq. (31) for the expressions of p_{11}, p_{12}, p_{22} that when the following condition is satisfied

$$c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)} > \tilde{c}_{44}^{(1)}c_{44}^{(2)}\in_{11}^{(1)}, \tag{44}$$

then the dislocation is always attracted to a traction-free and dielectrically perfect interface, i.e.,

$$F_y = -\frac{p_{11}b^2 + 2p_{12}b\Delta\phi + p_{22}\Delta\phi^2}{4\pi\delta} < 0. \tag{45}$$

3.2. Electroded Kelvin-type viscoelastic interface

The continuity condition Eq. (8) on an electroded Kelvin-type viscoelastic interface can be equivalently expressed in terms of $g_1(z,t)$, $g_2(z,t)$ defined in the upper half-plane and $h_1(z,t)$, $h_2(z,t)$ defined in the lower half-plane as follows

$$\begin{aligned}
&c_{44}^{(1)}[g_1^+(x,t)+\bar{g}_1^-(x,t)] + e_{15}^{(1)}[g_2^+(x,t)+\bar{g}_2^-(x,t)] = c_{44}^{(2)}[h_1^-(x,t)+\bar{h}_1^+(x,t)] + e_{15}^{(2)}[h_2^-(x,t)+\bar{h}_2^+(x,t)],\\
&g_2^+(x,t)-\bar{g}_2^-(x,t)=0,\\
&h_2^-(x,t)-\bar{h}_2^+(x,t)=0,\\
&k[g_1^+(x,t)-\bar{g}_1^-(x,t)-h_1^-(x,t)+\bar{h}_1^+(x,t)] + \eta\frac{\partial}{\partial t}[g_1^+(x,t)-\bar{g}_1^-(x,t)-h_1^-(x,t)+\bar{h}_1^+(x,t)]\\
&\quad = \mathrm{i}c_{44}^{(2)}\frac{\partial}{\partial x}[h_1^-(x,t)+\bar{h}_1^+(x,t)] + \mathrm{i}e_{15}^{(2)}\frac{\partial}{\partial x}[h_2^-(x,t)+\bar{h}_2^+(x,t)]. \quad \text{on } y=0.
\end{aligned} \tag{46}$$

It follows from $(46)_{1-3}$ that the following relationships hold in the upper half-plane

$$\begin{aligned}
&g_1(z,t) = \frac{c_{44}^{(2)}}{c_{44}^{(1)}}\bar{h}_1(z,t) + g_{10}(z) - \bar{g}_{10}(z) - \frac{2e_{15}^{(1)}}{c_{44}^{(1)}}\bar{g}_{20}(z),\\
&g_2(z,t) = g_{20}(z) + \bar{g}_{20}(z),\\
&\bar{h}_2(z,t) = 0,
\end{aligned} \tag{47}$$

whilst the following relationships hold in the lower half-plane

$$\begin{aligned}
&\bar{g}_1(z,t) = \frac{c_{44}^{(2)}}{c_{44}^{(1)}}h_1(z,t) + \bar{g}_{10}(z) - g_{10}(z) - \frac{2e_{15}^{(1)}}{c_{44}^{(1)}}g_{20}(z),\\
&\bar{g}_2(z,t) = g_{20}(z) + \bar{g}_{20}(z),\\
&h_2(z,t) = 0.
\end{aligned} \tag{48}$$

Substituting Eqs. (47) and (48) into Eq. $(46)_4$, we arrive at the following condition on the interface

$$\begin{aligned}
&k\frac{c_{44}^{(1)}+c_{44}^{(2)}}{c_{44}^{(1)}}\bar{h}_1^+(x,t) - \mathrm{i}c_{44}^{(2)}\frac{\partial\bar{h}_1^+(x,t)}{\partial x} + \eta\frac{c_{44}^{(1)}+c_{44}^{(2)}}{c_{44}^{(1)}}\frac{\partial\bar{h}_1^+(x,t)}{\partial t} - 2k\bar{g}_{10}(x) - \frac{2ke_{15}^{(1)}}{c_{44}^{(1)}}\bar{g}_{20}(x)\\
&\quad = k\frac{c_{44}^{(1)}+c_{44}^{(2)}}{c_{44}^{(1)}}h_1^-(x,t) + \mathrm{i}c_{44}^{(2)}\frac{\partial h_1^-(x,t)}{\partial x} + \eta\frac{c_{44}^{(1)}+c_{44}^{(2)}}{c_{44}^{(1)}}\frac{\partial h_1^-(x,t)}{\partial t} - 2kg_{10}(x)\\
&\qquad - \frac{2ke_{15}^{(1)}}{c_{44}^{(1)}}g_{20}(x), \quad \text{on } y=0.
\end{aligned} \tag{49}$$

It is apparent that the left-hand side of Eq. (49) is analytic in the upper half-plane, whilst the right-hand side of Eq. (49) is analytic in the lower half-plane. Consequently the continuity condition in Eq. (49) implies that the left- and right-hand sides of Eq. (49) are identically zero in the upper and lower half-planes, respectively. It follows that

$$-\mathrm{i}\chi h_1(z,t)+\frac{\partial h_1(z,t)}{\partial z}-\mathrm{i}\gamma\frac{\partial h_1(z,t)}{\partial t}=-\frac{\mathrm{i}\chi(c_{44}^{(1)}b+e_{15}^{(1)}\Delta\phi)}{\pi(c_{44}^{(1)}+c_{44}^{(2)})}\ln(z-\mathrm{i}\delta),\quad y\leqslant 0,\tag{50}$$

where

$$\chi=k\frac{c_{44}^{(1)}+c_{44}^{(2)}}{c_{44}^{(1)}c_{44}^{(2)}},\quad \gamma=\eta\frac{c_{44}^{(1)}+c_{44}^{(2)}}{c_{44}^{(1)}c_{44}^{(2)}}.\tag{51}$$

It is found from Eq. (51) that the two parameters χ and γ for the electroded Kelvin-type viscoelastic interface are independent of the piezoelectric and dielectric constants of the piezoelectric bimaterial. Since at $t=0$ when the piezoelectric screw dislocation is just introduced into the upper piezoelectric half-plane, the displacement across the interface has no time to experience any jump due to the dashpot, the displacement is, therefore, continuous across the interface at $t=0$ (i.e., the interface is mechanically perfect when $t=0$). In other words, the following initial condition for $\frac{\partial h_1(z,t)}{\partial z}$ holds

$$\frac{\partial h_1(z,0)}{\partial z}=\frac{(c_{44}^{(1)}b+e_{15}^{(1)}\Delta\phi)}{\pi(c_{44}^{(1)}+c_{44}^{(2)})(z-\mathrm{i}\delta)}.\tag{52}$$

When $t\to\infty$, the interface should be at a steady state and there is no time effect. In this case it follows from Eq. (50) that

$$-\mathrm{i}\chi h_1(z,\infty)+\frac{\partial h_1(z,\infty)}{\partial z}=-\frac{\mathrm{i}\chi(c_{44}^{(1)}b+e_{15}^{(1)}\Delta\phi)}{\pi(c_{44}^{(1)}+c_{44}^{(2)})}\ln(z-\mathrm{i}\delta),\tag{53}$$

with its solution being given by

$$\frac{\partial h_1(z,\infty)}{\partial z}=\frac{\mathrm{i}\chi(c_{44}^{(1)}b+e_{15}^{(1)}\Delta\phi)}{\pi(c_{44}^{(1)}+c_{44}^{(2)})}\exp\left[\chi(z-\mathrm{i}\delta)\right]E_1[\chi(z-\mathrm{i}\delta)].\tag{54}$$

In addition,

$$\frac{\partial h_1(z,t)}{\partial z}\to 0\quad\text{as } z\to\infty\tag{55}$$

due to the fact that at the far field the stresses and electric displacements should approach zero. In view of the initial state Eq. (52), the steady state Eq. (54) and the far-field condition Eq. (55), solution to Eq. (50) can be easily found to be

$$\begin{aligned}\frac{\partial h_1(z,t)}{\partial z}=&\frac{\mathrm{i}\chi(c_{44}^{(1)}b+e_{15}^{(1)}\Delta\phi)}{\pi(c_{44}^{(1)}+c_{44}^{(2)})}\exp[\mathrm{i}\chi(z-\mathrm{i}\delta)]\{E_1[\mathrm{i}\chi(z-\mathrm{i}\delta)]-E_1[\mathrm{i}\chi(z-\mathrm{i}\delta-\mathrm{i}t/\gamma)]\}\\&+\frac{(c_{44}^{(1)}b+e_{15}^{(1)}\Delta\phi)\exp(-\chi t/\gamma)}{\pi(c_{44}^{(1)}+c_{44}^{(2)})(z-\mathrm{i}\delta-\mathrm{i}t/\gamma)}.\end{aligned}\tag{56}$$

Then it follows from Eq. (47) that

$$\begin{aligned}\frac{\partial g_1(z,t)}{\partial z}=&-\frac{\mathrm{i}\chi c_{44}^{(2)}(c_{44}^{(1)}b+e_{15}^{(1)}\Delta\phi)}{\pi c_{44}^{(1)}(c_{44}^{(1)}+c_{44}^{(2)})}\exp\left[-\mathrm{i}\chi(z+\mathrm{i}\delta)\right]\{E_1[-\mathrm{i}\chi(z+\mathrm{i}\delta)]-E_1[-\mathrm{i}\chi(z+\mathrm{i}\delta+\mathrm{i}t/\gamma)]\}\\&+\frac{c_{44}^{(2)}(c_{44}^{(1)}b+e_{15}^{(1)}\Delta\phi)\exp(-\chi t/\gamma)}{\pi c_{44}^{(1)}(c_{44}^{(1)}+c_{44}^{(2)})(z+\mathrm{i}\delta+\mathrm{i}t/\gamma)}-\frac{c_{44}^{(1)}b+2e_{15}^{(1)}\Delta\phi}{2\pi c_{44}^{(1)}(z+\mathrm{i}\delta)}+\frac{b}{2\pi(z-\mathrm{i}\delta)},\end{aligned}\tag{57}$$

and

$$\frac{\partial g_2(z,t)}{\partial z}=\frac{\Delta\phi}{2\pi(z+\mathrm{i}\delta)}+\frac{\Delta\phi}{2\pi(z-\mathrm{i}\delta)}.\tag{58}$$

When $k = 0$ (or $\chi \to 0$) for a viscous interface, it can be deduced from Eqs. (56) and (57) that the expressions of $\frac{\partial h_1(z,t)}{\partial z}$ and $\frac{\partial g_1(z,t)}{\partial z}$ for an electroded viscous interface are given by

$$\frac{\partial h_1(z,t)}{\partial z} = \frac{c_{44}^{(1)} b + e_{15}^{(1)} \Delta\phi}{\pi(c_{44}^{(1)} + c_{44}^{(2)})(z - \mathrm{i}\delta - \mathrm{i}t/\gamma)}, \tag{59}$$

$$\frac{\partial g_1(z,t)}{\partial z} = \frac{c_{44}^{(2)}(c_{44}^{(1)} b + e_{15}^{(1)} \Delta\phi)}{\pi c_{44}^{(1)}(c_{44}^{(1)} + c_{44}^{(2)})(z + \mathrm{i}\delta + \mathrm{i}t/\gamma)} - \frac{c_{44}^{(1)} b + 2e_{15}^{(1)} \Delta\phi}{2\pi c_{44}^{(1)}(z + \mathrm{i}\delta)} + \frac{b}{2\pi(z - \mathrm{i}\delta)}. \tag{60}$$

With the expressions of $\frac{\partial g_1(z,t)}{\partial z}$ and $\frac{\partial g_2(z,t)}{\partial z}$ defined in the upper piezoelectric half-plane, and $\frac{\partial h_1(z,t)}{\partial z}$ and $\frac{\partial h_2(z,t)}{\partial z}$ defined in the lower piezoelectric half-plane for an electroded Kelvin-type viscoelastic interface, the distributions of strains, stresses, electric fields and electric displacements in the two half-planes can then be obtained from Eq. (5).

By using the Peach-Koehler formulation, the image force acting on the screw dislocation due to its interaction with the electroded Kelvin-type viscoelastic interface can be also derived to be

$$\begin{aligned} F_y = &-\frac{(c_{44}^{(1)} b + e_{15}^{(1)} \Delta\phi)^2 + \tilde{c}_{44}^{(1)} \in_{11}^{(1)} \Delta\phi^2}{4\pi c_{44}^{(1)} \delta} \\ &+ \frac{c_{44}^{(2)}(c_{44}^{(1)} b + e_{15}^{(1)} \Delta\phi)^2}{2\pi c_{44}^{(1)}(c_{44}^{(1)} + c_{44}^{(2)})\delta} \left\{ g(\lambda) + \left(1 + \frac{\tilde{t}}{2}\right)^{-1} \exp(-\lambda\tilde{t}) \left[1 - g\left(\lambda + \frac{\lambda\tilde{t}}{2}\right)\right] \right\}, \\ F_x = &\ 0, \end{aligned} \tag{61}$$

where $\lambda = \delta\chi$ and $t_0 = \delta\gamma$ are, respectively, the interface "rigidity" and the relaxation time for the electroded Kelvin-type viscoelastic interface, and $\tilde{t} = t/t_0$. It is interesting from the above expression that the image force on the screw dislocation due to its interaction with the electroded Kelvin-type viscoelastic interface is independent of the piezoelectric and dielectric properties of the lower half-plane, which is free of the screw dislocation. Somewhat surprising is the case when $\Delta\phi = 0$: Eq. (61) will be reduced to the result of Fan and Wang (2003b) for a screw dislocation interacting with a Kelvin-type viscoelastic interface between two *elastic* half-planes. In other words if an *elastic* dislocation interacts with an electroded Kelvin-type viscoelastic interface between two piezoelectric half-planes, then the piezoelectric and dielectric properties of both piezoelectric half-planes have no influence on the mobility of the dislocation!

For a viscous interface we have $\lambda = 0$ and $g(0) = 0$, then it follows from Eq. (61) that

$$F_y = -\frac{(c_{44}^{(1)} b + e_{15}^{(1)} \Delta\phi)^2 + \tilde{c}_{44}^{(1)} \in_{11}^{(1)} \Delta\phi^2}{4\pi c_{44}^{(1)} \delta} + \frac{c_{44}^{(2)}(c_{44}^{(1)} b + e_{15}^{(1)} \Delta\phi)^2}{2\pi c_{44}^{(1)}(c_{44}^{(1)} + c_{44}^{(2)})\delta} \left(1 + \frac{\tilde{t}}{2}\right)^{-1}. \tag{62}$$

When $\tilde{t} \to \infty$, this expression is reduced to

$$F_y = -\frac{(c_{44}^{(1)} b + e_{15}^{(1)} \Delta\phi)^2 + \tilde{c}_{44}^{(1)} \in_{11}^{(1)} \Delta\phi^2}{4\pi c_{44}^{(1)} \delta} < 0, \tag{63}$$

which is the image force on a screw dislocation interacting with a traction-free and electroded surface.

It is found from Eq. (61) that if b and $\Delta\phi$ satisfy the relation $b = -\frac{e_{15}^{(1)} \Delta\phi}{c_{44}^{(1)}}$, Eq. (61) then is reduced to Eq. (63). In this special case the image force on the dislocation due to its interaction with an electroded Kelvin-type viscoelastic interface is the same as that on the dislocation interacting with a traction-free and electroded surface.

Finally, it is of interest to compare the values of the interface "rigidity" $\lambda = \delta\chi$ and the relaxation time $t_0 = \delta\gamma$ for the unelectroded and electroded cases. Due to the fact that the following inequality holds

$$\frac{(\tilde{c}_{44}^{(1)}+c_{44}^{(2)})\in_{11}^{(1)}+(c_{44}^{(1)}+\tilde{c}_{44}^{(2)})\in_{11}^{(2)}+2e_{15}^{(1)}e_{15}^{(2)}}{\tilde{c}_{44}^{(1)}c_{44}^{(2)}\in_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)}}\leqslant\frac{c_{44}^{(1)}+c_{44}^{(2)}}{c_{44}^{(1)}c_{44}^{(2)}}, \tag{64}$$

then the values of λ and t_0 for the unelectroded case are smaller than the corresponding ones for the electroded case.

4. Conclusions

A detailed theoretical analysis is presented for the interaction between a screw dislocation and a Kelvin-type viscoelastic interface bonding two transversely isotropic piezoelectric half-planes. Both the unelectroded and electroded cases are investigated. The analytical solutions are obtained by virtue of the complex variable method. The image force acting on the piezoelectric screw dislocation is obtained. For an unelectroded interface, the image force can be completely determined by the interface "rigidity" λ, the relaxation time t_0 and the six coefficients p_{11}, p_{12}, p_{22} and q_{11}, q_{12}, q_{22} which are related to the material constants of the two piezoelectric half-planes. For an electroded interface, the image force is independent of the piezoelectric and dielectric properties of the lower half-plane which is free of the screw dislocation. When the viscous effect of the interface is ignored ($t_0 = 0$), our results reduce to those for a linear spring elastic interface. On the other hand when the elastic effect of the interface is ignored ($\lambda = 0$), our results reduce to those for a viscous interface. Finally it is mentioned that even though one could also derive the partial differential equation for a piezoelectric screw dislocation interacting with a Maxwell-type viscoelastic interface, it would be difficult to find the analytical solution, as we presented here for the Kelvin-type viscoelastic interface case. Similar to Ang and Fan (2004), a boundary integral method can also be proposed for the numerical solution of a quasi-static antiplane problem involving a piezoelectric bimaterial with an imperfect and viscoelastic interface.

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